PDF Evolution of a Pulse Train: An Adaptive Spline and Characteristic Function Approach

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Abstract

One important method for characterizing random phenomena is via the evolution of the probability density function. For the general case, where the random phenomena are characterized by a stochastic differential equation, a partial differential equation can be specified for the probability density function, $f_{X(t)}$; the solution of which is problematic as high order integration is required.

An important class of random phenomena can be modelled directly, without reference to the underlying dynamics, and according to

$$X(\Omega, t) = \sum_{k=1}^{N(t)} A_k h(t - \Gamma_k) \qquad \Omega = ((A_1, \Gamma_1), \ldots, (A_{N(t)}, \Gamma_{N(t)})) \tag{1}$$

where A_k and Γ_k are random variables that define, respectively, the amplitude and delay of the kth pulse function h. The model defines, depending on the nature of A_k, Γ_k and h, the random walk, the Poisson point process, the Poisson counting process, shot noise, generalized shot noise, the random telegraph signal, jittered pulse trains etc.

For the independent random variable case, the probability density function can be specified in terms of a N fold convolution. For this case, the usual approach is to use the characteristic function, or the Laplace transform, which converts the N fold convolution into a N fold multiplication followed by the use of an inverse Fourier or Laplace transform. For a relatively few cases (e.g. the Gaussian, Gamma, exponential distribution cases) where the inverse transforms exist, the analysis is straightforward. For the general case, approximation methods, e.g. via the use of Padé approximations, can be used to specify an approximation to the characteristic/Laplace transform function. Such approaches generally yield an approximation of the characteristic/Laplace transform over a limited range. In general, a large approximation range is required for accurate probability density function approximation.

In this presentation it will be shown, using the prototypical case of a transient pulse train, that determining the probability density function evolution, for the case where inverse transforms do not exist, can be achieved with relatively low errors if the characteristic function, to the require power, is dynamically approximated by spline functions. Simulation results justify the theory.

About the Speaker

Roy Howard holds an Adjunct Senior Research Fellow position in the School of Electrical Engineering and Computing at Curtin University, Perth, Australia and is a regular visitor to the Signal Processing Group, Technische Universität Darmstadt, Darmstadt, Germany. He has received a BE and Ph.D in Electrical Engineering from the University of Western Australia, Perth, Australia, respectively, in 1982 and 1988. In 1995 he received a BA from the same University with majors in Philosophy and Mathematics.

His expertise and current research interests include signal theory, modelling of random phenomena and low noise electronic design. He is the author of: ‘Principles of Random Signal Analysis and Low Noise Design: The Power Spectral Density and its Applications’, Wiley, 2002 and ‘A Signal Theoretic Introduction to Random Processes’, Wiley 2015.