Introduction to Robust Signal Processing

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Outline

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Introduction

Introduction

Motivation

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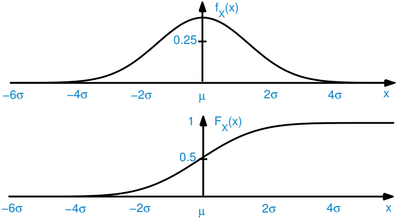
Classical Theoretical Approach to Estimation

- strong and precise assumptions
- e.g., estimators, detectors or filters optimal under nominal distribution



Optimality only achieved when assumptions hold exactly.

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Real World Data in Signal Processing

- in many cases Gaussian assumption well justified
- measurement campaigns confirmed impulsive (heavy-tailed) noise, [4] e.g. in
  - outdoor and indoor mobile communication channels
  - radar and sonar systems
  - biomedical sensor (array) measurements, e.g. magnetic resonance imaging (MRI)
- outliers in the measurements, e.g. in
  - geolocation position estimation and tracking (NLOS)
  - short-term load forecasting
  - motion artifacts for portable medical devices

Performance of optimal procedures may deteriorate significantly, even for minor departures from assumed model.

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Gaussianity

- Assume the noise is distributed according to $\mathcal{N}(0, \sigma^2)$. Then, the 'optimal' parametric estimator for μ is the sample mean (maximum likelihood estimator).
- The Gaussian assumption was introduced by Gauß in 1797. It was the distribution which resulted in the sample mean as the optimal estimator for the mean.

Gaussianity

'Everyone believed in the normal distribution, the mathematicians because they thought it was an experimental fact, the experimenters because they thought it was a mathematical theorem.'

'Normality is a myth; there never has, and never will be, a normal distribution', *[R. C. Geary 1947]*

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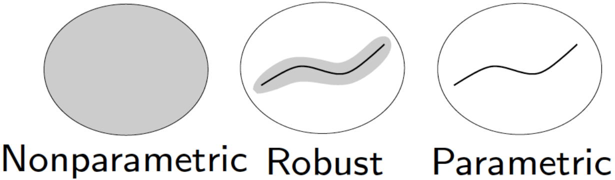
Applicability of Robust Methods

	Nonparametric	Robust	Parametric
Description	Model specified in terms of general properties	Parametric model allowing for deviations	Model completely specified by several parameters
Ideal Performance	Mediocre/Satisfactory	Good	Very Good/Excellent
Range of Validity	Large	Medium	Small



Robust theory is the most appropriate approach to solving real-life problems.

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Detecting Outliers

- Why not simply manually discard the outliers? Or find a simple rejection rule?
  - Large size of the data sets.
  - With an increasing dimension of the data, simple robust methods, based on outlier rejection, no longer work.
- Modern, sophisticated robust statistics aim at
  1. offering protection against complete breakdown (disasters) in performance due to outliers,
  2. reducing the loss in efficiency when deviations from the nominal statistical model occur.
- The concept of *optimal robustness* can be defined in different ways and this leads to different robust estimators.

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Detecting Outliers

- An outlier is tightly linked to the considered assumptions or model.
- an observation that is flagged as outlying relative to some model, may not be deemed to be outlying relative to another model.

Difficulty to detect outliers varies depending on the situation:

- 1 and 2 dimensions: relatively easy to detect by diagnostic methods, sometimes it is even possible via visual inspection
- Higher dimensions: Harder but feasible using diagnostic methods, though impossible via visual inspection
- Correlated signals or time series: very challenging

Introduction

Robustness: Intuitive Definitions

Robustness: The Bridge Analogy

movie source: archive.org

- [Hampel *et al.*, 2000]:

  'Robustness theory is the stability theory of statistical procedures.'

- [Huber *et al.*, 2009]:

  'Robustness signifies insensitivity to small deviations from the assumptions.'

Aims of Robust Methods

Approximate Quotes by Huber and Hampel

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Aim 1: Near Optimality [Huber-09]:

‘The procedure should behave “reasonably well” at the assumed model.’

Aims of Robust Methods

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Aim 2: Qualitative Robustness [Hampel-85]:

‘The effect of an erroneous observation, even if it takes an arbitrary value, should not have a large impact on the method.’

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’Somewhat larger deviations from the model should not cause a catastrophe.’

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How can I quantify, if my estimator fulfills these aims?

Measuring Robustness

Intuitive Definitions

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Measure 1: Relative Efficiency

- increase in the variance (σ^2) of the estimates *at the assumed model* compared to the optimal method

Measuring Robustness

Intuitive Definitions

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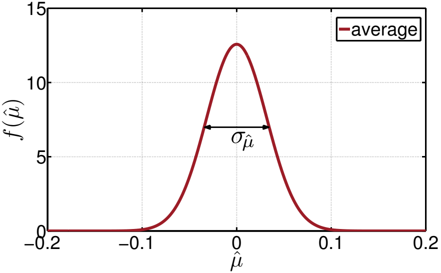

optimal estimator of the mean of a zero-mean Gaussian random variable

Measuring Robustness

Intuitive Definitions

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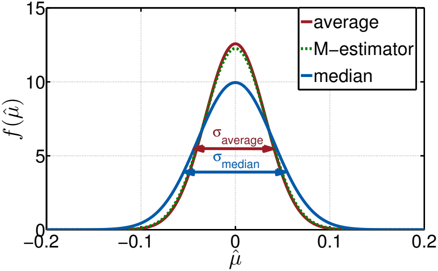


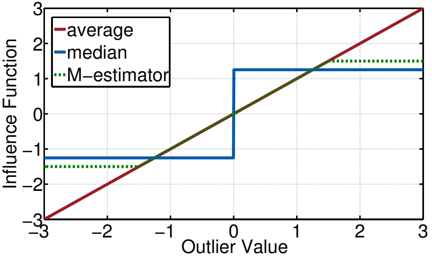
robust estimators of the mean of a zero-mean Gaussian random variable

Measuring Robustness

Intuitive Definitions

Measure 2: Influence Function (IF)

- bias impact of an infinitesimal contamination on the estimator, standardized by the fraction of contamination



bounded and continuous IF $\rightarrow$ qualitative robustness

Measuring Robustness

Intuitive Definitions

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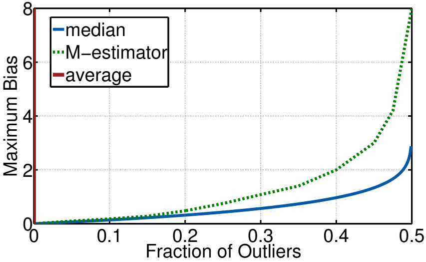
Measure 3: Maximum Bias Curve (MBC)

- maximum possible bias of an estimator plotted over the fraction of outliers



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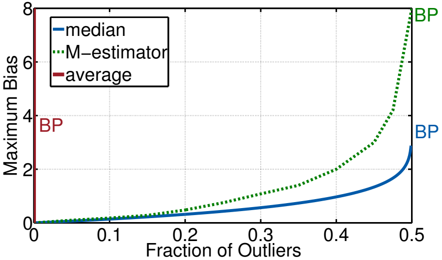

Measure 4: Breakdown Point (BP)

- maximal fraction of outliers that an estimator can handle ($0 \leq \text{BP} \leq 50\%$)

Basic Concepts: Estimating the Mean

Basic Concepts

Robustifying the Location Model

Location Model

$$X_n = \mu + V_n$$

- X_n: observable process
- μ: “true” (unknown) value
- V_n: random error process with probability distribution function $F_V(v)$

Task: estimate μ, given i.i.d. observations x_n, $n = 1, \ldots, N$.

- fundamental task in many statistical and engineering problems
- finds typical or central value that best describes the data

Most commonly used estimator: sample mean (average)

Is this a reliable/optimal/robust estimator?

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Maximum Likelihood (ML) Estimate of μ

$$\widehat{\mu}_{ML} = \arg\max_{\mu} \sum_{n=1}^{N} \log f_X(x_n|\mu)$$

- $\widehat{\mu}_{ML}$ solves:

$$\sum_{n=1}^{N} \psi(x_n - \widehat{\mu}_{ML}) = 0,$$

where

$$\psi(x) = -f_X'(x)/f_X(x).$$

Maximum Likelihood (ML) Estimate of μ

$$\widehat{\mu}_{ML} = \arg\max_{\mu} \sum_{n=1}^{N} \log f_X(x_n|\mu)$$

- If $F_X(x)$ is Gaussian

$$\sum_{n=1}^{N} (x_n - \widehat{\mu}_{ML}) = 0,$$

where

$$\psi(x) = -f'_X(x)/f_X(x) = x \ \rightarrow \text{sample mean}$$

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Maximum Likelihood (ML) Estimate of μ

$$\widehat{\mu}_{ML} = \arg\max_{\mu} \sum_{n=1}^{N} \log f_X(x_n|\mu)$$

- If $F_X(x)$ is Laplacian

$$\sum_{n=1}^{N} \text{sign}(x_n - \widehat{\mu}_{ML}) = 0,$$

where

$$\psi(x) = -f'_X(x)/f_X(x) = \text{sign}(x) \quad \rightarrow \text{sample median}$$

How about robustness of ML estimators?

Basic Concepts

Breakdown Point: Practitioners Definition

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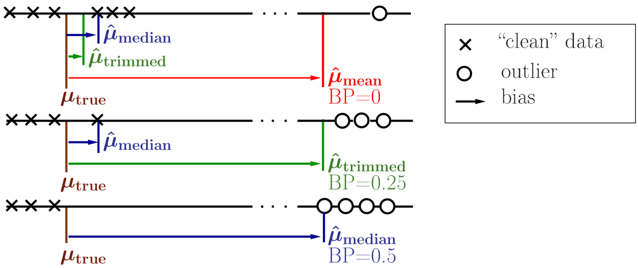
Breakdown Point (BP)

movie source: archive.org

Practitioners Definition: Percentage of data that can be replaced by arbitrary values without driving the bias of the estimator to infinity.

- very simple quantitative concept, independent of probabilistic notions
- most useful in small sample situations [Huber *et al.* 2009]

Bias and Breakdown Point



$\hat{\mu}_{\text{mean}}$: sample mean

$\hat{\mu}_{\text{trimmed}}$: α-trimmed mean (α = 25%)

$\hat{\mu}_{\text{median}}$: sample median

Basic Concepts

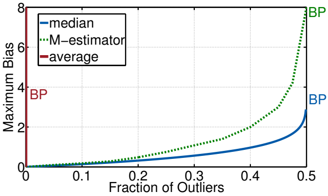
Link between BP and Maximum Bias Curve

Maximum Bias Curve (MBC)

- displays the maximum possible bias for a given percentage of outliers

$$\text{MBC}(\epsilon, \theta) = \max\left\{\left|\hat{b}_\theta(F, \theta)\right| : F \in \mathcal{F}_{\epsilon,\theta}\right\}$$

  $\mathcal{F}_{\epsilon,\theta} = \{(1-\epsilon)F_\theta + \epsilon G\}$: ϵ-neighbourhood of distributions around the nominal distribution F_θ with G being an arbitrary contamintaing distribution
- practical tool to assess the breakdown point, e.g. mean, median and M-estimator



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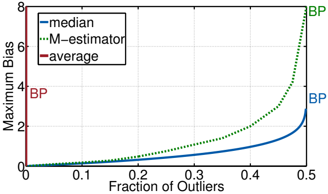
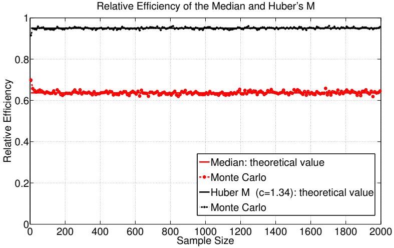


So why doesn't everyone simply use the median?

Efficiency when F is standard normal



- relative efficiency of median is low: Eff[$\hat{\mu}_{\text{median}}$] = $2/\pi$
- M-estimators: BP=0.5 and Eff[$\hat{\mu}_M$] = 0.95

$\rightarrow$ What are M-estimators?

Basic Concepts

M-Estimation

M-estimate of μ [Huber 1964]

Key Idea: replace $\log f_X(x)$ from $\widehat{\mu}_{ML}$ by $\rho(x)$

$$\widehat{\mu}_M = \arg\max_{\mu} \sum_{n=1}^{N} \rho(x_n - \mu)$$

$\widehat{\mu}_M$ solves:

$$\sum_{n=1}^{N} \psi(x_n - \widehat{\mu}_M) = 0,$$

where

$$\psi(x) = \rho'(x).$$

- $\rightarrow$ includes all MLE
- $\rightarrow$ $\psi(x)$ bounded $\rightarrow$ BP = 0.5

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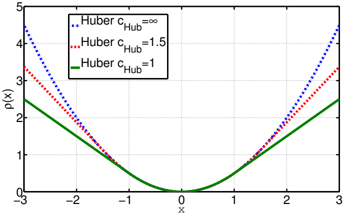
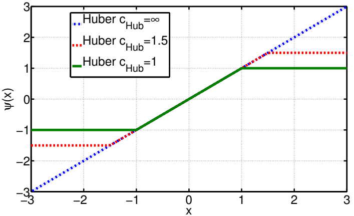
Example: Huber's Robust M-estimator - monotone $\psi(x)$

$$\rho(x) = \begin{cases} \frac{1}{2}x^2 & |x| \leq c_{Hub} \\ c_{Hub}|x| - \frac{1}{2}c_{Hub}^2 & |x| > c_{Hub} \end{cases}$$

$$\psi(x) = \begin{cases} x & |x| \leq c_{Hub} \\ c_{Hub}\text{sign}(x) & |x| > c_{Hub} \end{cases}$$



(a) ρ-function



(b) score function ψ

Basic Concepts
M-Estimation: Example

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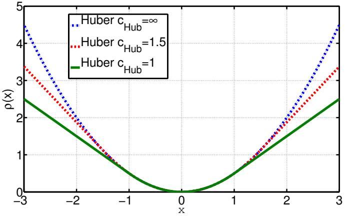

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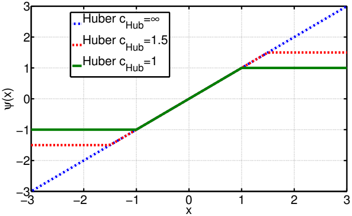

(b) score function ψ

Remark 1: c_{Hub} = const. $\cdot\, \hat{\sigma}_x$, or equivalently $\psi(\frac{x_n - \widehat{\mu}_M}{\hat{\sigma}_x})$ and c_{Hub} = const., where $\hat{\sigma}_x$ is a robust scale estimate.

Basic Concepts

M-Estimation: Example

Example: Huber's Robust M-estimator - monotone $\psi(x)$

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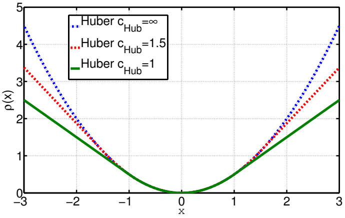

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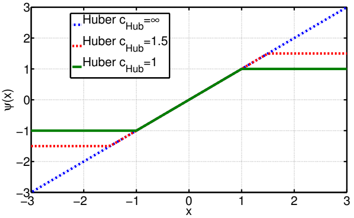

(b) score function ψ

Remark 2: Score function of M-estimators is directly related to the influence function $\rightarrow$ robustness properties easy to modify based on ψ!

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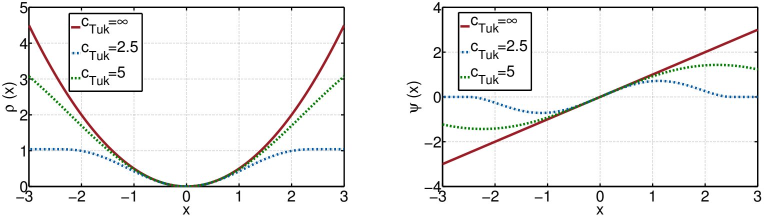
Example: Tukey's Robust M-estimator - redescending $\psi(x)$

$$\rho(x) = \begin{cases} \frac{x^2}{2} - \frac{x^4}{2c_{\text{Tuk}}^2} + \frac{x^6}{6c_{\text{Tuk}}^4} & |x| \leq c_{\text{Tuk}} \\ \frac{c_{\text{Tuk}}^2}{6} & |x| > c_{\text{Tuk}} \end{cases}$$

$$\psi(x) = \begin{cases} x - 2\frac{x^3}{c_{\text{Tuk}}^2} + \frac{x^5}{c_{\text{Tuk}}^4} & |x| \leq c_{\text{Tuk}} \\ 0 & |x| > c_{\text{Tuk}}. \end{cases}$$



(a) ρ-function

(b) score function ψ

Influence Function (IF)

movie source: archive.org

- IF measures stability of the estimator against:
  1. changing a tiny fraction of the data drastically (outlier)
  2. changing a large fraction marginally (rounding)

Bounded and continuous IF $\rightarrow$ stability over an entire family of distributions.

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Influence Function (IF): Definition and Properties

When the limit exists, the asymptotic IF, is defined by

$$\mathrm{IF}(z;\hat{\theta},F_\theta)=\lim_{\epsilon\to 0}\frac{\hat{\theta}_\infty(G)-\hat{\theta}_\infty(F_\theta)}{\epsilon}=\left[\frac{\partial\hat{\theta}_\infty(G)}{\partial\epsilon}\right]_{\epsilon=0},$$

- First derivative of the functional version of an estimator $\hat{\theta}$ at a nominal distribution F_θ
- ϵ: fraction of contamination.
- $\hat{\theta}_\infty(F_\theta)$: asymptotic value of the estimator when the data follows the nominal distribution F_θ
- $\hat{\theta}_\infty(G)$: asymptotic value of the estimator when the data follows the Tukey-Huber Contamination model $G=(1-\epsilon)F_\theta+\epsilon\Delta_z$
- Δ_z: point-mass probability on z.

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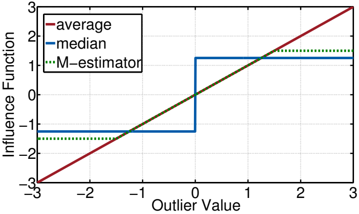
- IF is depicted with respect to z, the position of the infinitesimal contamination
- Desirable properties of the IF: boundedness and continuity
  - boundedness $\rightarrow$ small fraction of contamination or outliers only has limited effect on the estimate
  - continuity $\rightarrow$ small changes in the data lead to small changes in the estimate

Boundedness and continuity: estimator is *qualitatively robust* against infinitesimal contamination

Basic Concepts

IF of Some Popular Estimators

Influence Function (IF) of Estimators of μ at $F_X(x) = \mathcal{N}(0, 1)$



- for M-estimators $\text{IF}_{\widehat{\mu}_M}(z, F_X(x)) \sim \psi(z)$
- of the above three, only Huber's M-estimator is qualitatively and quantitatively robust

Algorithm: Location M-Estimation With Previously Computed Scale

Step 1. estimate $\hat{\sigma}$ and initial $\hat{\mu}_o$, e.g. with

$$\hat{\sigma}_{mad}(\mathbf{x}) = 1.483 \cdot \text{median}(|\mathbf{x} - \text{median}(\mathbf{x})|)$$

and

$$\hat{\mu}_o(\mathbf{x}) = \text{median}(\mathbf{x})$$

Step 2. while $\frac{|\hat{\mu}_{k+1} - \hat{\mu}_k|}{\hat{\sigma}} < \xi$, do

$$w_{kn} = W\left(\frac{x_n - \hat{\mu}_k}{\hat{\sigma}}\right) \quad \text{and} \quad \hat{\mu}_{k+1} = \frac{\sum_{n=1}^{N} w_{kn} x_n}{\sum_{n=1}^{N} w_{kn}}$$

$$k \leftarrow k + 1$$

$W(x)$ is given by

$$W(x) = \begin{cases} \psi(x)/x, & \text{if } x \neq 0 \\ \psi'(0), & \text{if } x = 0. \end{cases}$$

For the location model, e.g. Huber's M-estimator fulfils

Aim 1: should behave “reasonably good” at the assumed model

- efficiency = 0.95 for $c_{Hub} = 1.34$

Aim 2: qualitative robustness

- IF bounded and continuous

Aim 3: quantitative robustness

- BP = 0.5

Basic Concepts: Estimating the Scale

Multiplicative Model: Random Variables With Density of Type

$$f_X(x) = \frac{1}{\sigma} g\left(\frac{x}{\sigma}\right), \quad \sigma > 0, \quad \text{e.g.}$$

- Gaussian distribution: $f = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
- Laplace distribution: $f = \frac{1}{2\sigma} e^{\frac{-|x-\mu|}{\sigma}}$

Task: Estimate σ given i.i.d. observations $x_n, n = 1, \dots, N$

- Engineering: σ nuissance parameter (e.g. in location estimtation, regression)
- larger $\sigma \rightarrow$ more spread distribution

Most commonly used estimator: sample standard deviation

Basic Concepts

Maximum Likelihood Scale Estimation

Maximum Likelihood (ML) Estimate of σ

$$\hat{\sigma}_{ML} = \underset{\sigma}{\operatorname{argmax}} \frac{1}{N} \prod_{n=1}^{N} f\left(\frac{x_n}{\sigma}\right)$$

taking logs and differentiating w.r.t. σ:

$$\frac{1}{N} \sum_{n=1}^{N} \rho_{ML}\left(\frac{x_n}{\hat{\sigma}}\right) = 1$$

- $\rho_{ML}(x) = x\psi(x)$
- $\psi(x) = -\frac{f'(x)}{f(x)}$

Basic Concepts

Maximum Likelihood Scale Estimation

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Maximum Likelihood (ML) Estimate of σ

$$\hat{\sigma}_{ML} = \underset{\sigma}{\operatorname{argmax}} \frac{1}{N} \prod_{n=1}^{N} f\left(\frac{x_n}{\sigma}\right)$$

- if F is standard Gaussian

$$\rho_{ML}(x) = x^2 \quad \rightarrow \quad \hat{\sigma}_{ML} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} x_n^2}$$

$\rightarrow$ $\hat{\sigma}_{ML}$: sample standard deviation

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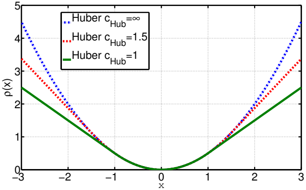
M-Estimate of σ

$$\frac{1}{N}\sum_{n=1}^{N}\rho\left(\frac{x_n}{\hat{\sigma}_M}\right)=\delta,$$

- $\rho(x)$: as discussed in the previous section, e.g. Huber's



- $0 < \delta < \rho(\infty)$: a positive constant

$\rho(x)$ bounded, continuous and quadratic near the origin

$\rightarrow$ robust and efficient estimates

$$\hat{\sigma}_M = \sqrt{\frac{1}{N\delta}\sum_{n=1}^{N} W\left(\frac{x_n}{\hat{\sigma}_M}\right) x_n^2}$$

- $W(x) = \begin{cases} \rho(x)/x^2 & x \neq 0 \\ \rho''(0) & x = 0 \end{cases}$

$\rightarrow \hat{\sigma}_M$: weighted root mean square estimate!

Basic Concepts

M-Estimation of Scale

(Standardized) Median Absolute Deviation (MAD)

$$\hat{\sigma}_{\text{MAD}}(\mathbf{x}) = 1.483\text{median}\{|\mathbf{x} - \text{median}(\mathbf{x})|\}$$

- $\mathbf{x} = (x_1, \dots, x_N)^T$
- $1.483 = 1/F^{-1}(3/4) \rightarrow$ Fisher consistency for standard Gaussian
- frequently used M-estimator
- BP = 50 %
- IF: smallest supremum of all M-estimators $\rightarrow$ most B-robust
- However: relative efficiency ≈ 0.367

Algorithm:

Step 1. start with initial $\hat{\sigma}_o$, e.g. $\hat{\sigma}_{\text{MAD}}(\mathbf{x})$

Step 2. while $\frac{\hat{\sigma}_{k+1}}{\hat{\sigma}} - 1 < \xi$, do

$$w_{kn} = W\left(\frac{x_n}{\hat{\sigma}_k}\right)$$

and

$$\hat{\sigma}_{k+1} = \sqrt{\frac{1}{N\delta} \sum_{n=1}^{N} w_{kn} x_n^2}$$

$k \leftarrow k + 1$

$W(x)$ is given by

$$W(x) = \begin{cases} \psi(x)/x, & \text{if } x \neq 0 \\ \psi'(0), & \text{if } x = 0. \end{cases}$$

Basic Concepts

Illustrating Example Location and Scale Estimation

Analysing Copper Content of Wholemeal Flour

en.wikipedia.org

24 measurements (in p.p.m.)

(2.20, 2.20, 2.40, 2.40, 2.50, 2.70,
2.80, 2.90, 3.03, 3.03, 3.10, 3.37,
3.40, 3.40, 3.40, 3.50, 3.60, 3.70,
3.70, 3.70, 3.70, 3.77, 5.28, 28.95)

outlier at 28.95

- measurement error?
- heavy tailed distribution?
- decimal error (true value 2.895)?

Basic Concepts

Illustrating Example Location and Scale Estimation

Analysing Copper Content of Wholemeal Flour

en.wikipedia.org

Location Estimates

- sample mean: 4.28
  (without the outlier 3.21)
- sample median: 3.38
  (without the outlier 3.37)
- M-estimate: 3.21
  (without the outlier 3.16)

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Analysing Copper Content of Wholemeal Flour

en.wikipedia.org

Scale Estimates

- sample standard deviation: 5.30 (without the outlier 0.69)
- normalized MAD: 0.53 (without the outlier 0.50)
- M-estimate: 0.63 (without the outlier 0.59)

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Basic Concepts: Robust Detection

Notation

N i.i.d. random variables $X_1, \dots, X_N$

Hypotheses $\mathcal{H}_0 : X_n \sim P_0, \quad \mathcal{H}_1 : X_n \sim P_1$

Likelihood ratio $z^N = \prod_{n=1}^{N} \frac{p_1(x_n)}{p_0(x_n)}$

Decision rule $\delta \in \{0, 1\}$

Error probabilities
$P_0[\delta = 1]$ type I
$P_1[\delta = 0]$ type II

Robust Detection

Fixed Sample Size Tests

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Notation

N i.i.d. random variables	$X_1, \dots, X_N$
Hypotheses	$\mathcal{H}_0 : X_n \sim P_0, \quad \mathcal{H}_1 : X_n \sim P_1$
Likelihood ratio	$z^N = \prod_{n=1}^{N} \frac{p_1(x_n)}{p_0(x_n)}$
Decision rule	$\delta \in \{0, 1\}$
Error probabilities	$P_0[\delta = 1]$ type I $P_1[\delta = 0]$ type II

- P_0 and P_1 not precisely known
- introduce an uncertainty model: $P_0, P_1 \quad \rightarrow \quad P_0 \in \mathcal{P}_0,\ P_1 \in \mathcal{P}_1$

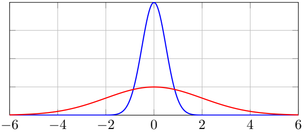
Robust Detection

Minimax Fixed Sample Tests

Examples of Uncertainty Models

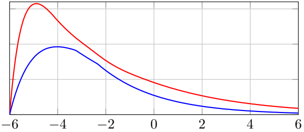
Outliers

$$p = (1 - \varepsilon) p_{\text{nom}} + \varepsilon h_{\text{outl}}$$



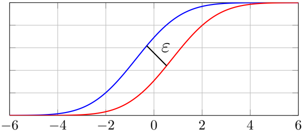
Density Band

$$p' \leq p \leq p''$$



Distance Tolerance

$$L(P_{\text{nom}}, P) \leq \varepsilon$$



Robust Detection

Minimax Fixed Sample Tests

Minimax Optimality

Use optimal test between least favorable pair of distributions $(Q_0, Q_1) \in \mathcal{P}_0 \times \mathcal{P}_1$

$$\prod_{n=1}^{N} \frac{q_1(x_n)}{q_0(x_n)} \lessgtr \lambda$$

Robust Detection

Minimax Fixed Sample Tests

Minimax Optimality

Use optimal test between least favorable pair of distributions $(Q_0, Q_1) \in \mathcal{P}_0 \times \mathcal{P}_1$

$$\prod_{n=1}^{N} \frac{q_1(x_n)}{q_0(x_n)} \lessgtr \lambda$$

The pair (Q_0, Q_1) is least favorable, if it jointly maximizes both error probabilities

$$Q_0[\delta = 1] \geq P_0[\delta = 1] \qquad \forall P_0 \in \mathcal{P}_0$$
$$Q_1[\delta = 0] \geq P_1[\delta = 0] \qquad \forall P_1 \in \mathcal{P}_1$$

for all $(P_0, P_1) \in \mathcal{P}_0 \times \mathcal{P}_1$.

Robust Detection

Minimax Fixed Sample Tests

Minimax Optimality

Use optimal test between least favorable pair of distributions $(Q_0, Q_1) \in \mathcal{P}_0 \times \mathcal{P}_1$

$$\prod_{n=1}^{N} \frac{q_1(x_n)}{q_0(x_n)} \lessgtr \lambda$$

The pair (Q_0, Q_1) is least favorable, if it jointly minimizes all f-divergences

$$\int f\left(\frac{q_1(x)}{q_0(x)}\right) q_0(x)\,\mathrm{d}x \;\leq\; \int f\left(\frac{p_1(x)}{p_0(x)}\right) p_0(x)\,\mathrm{d}x$$

for all $(P_0, P_1) \in \mathcal{P}_0 \times \mathcal{P}_1$ and for all convex functions f.

Huber, P. J. and Strassen, V. (1973). Minimax Tests and the Neyman–Pearson Lemma for Capacities. *Annals of Statistics* 1: 251–263.

Robust Detection

Minimax Fixed Sample Tests

Minimax Optimality

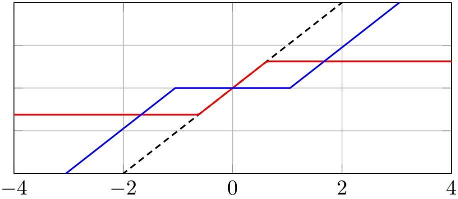
Use optimal test between least favorable pair of distributions $(Q_0, Q_1) \in \mathcal{P}_0 \times \mathcal{P}_1$

$$\prod_{n=1}^{N} \frac{q_1(x_n)}{q_0(x_n)} \lessgtr \lambda$$

Example: Detect shift in standard normal distribution

Log-likelihood ratio under

- nominal model
- 10% outlier rate
- density corridor



Where We Are Today: Advanced Robust Estimation Tasks in Engineering

Problems in Communication, e.g.

Direction of Arrival Estimation [Tsakalides *et al.*,1996],[Liu *et al.*,2001], [Visuri *et al.*,2001],[Swami *et al.*,2002],[Ollila *et al.*,2003], [Lim *et al.*,2009], [Sharif *et al.*,2013]

image source: www.istockphoto.com

Where are we Today?

Advanced Robust Estimation Tasks in Engineering

Problems in Communication, e.g.

Mobile Positioning in Wireless Networks [Kumar *et al.*,2009], [Guvenc *et al.*,2009],[Hammes *et al.*,2011], [Yin *et al.*,2013]

image source: www.istockphoto.com

Problems in Communication, e.g.

Multiuser Detection [Wang *et al.*,1999],[Zoubir *et al.*,2002], [Poor *et al.*,2002],[Chen *et al.*,2005],[Pham *et al.*,2006],[Kumar *et al.*,2009]

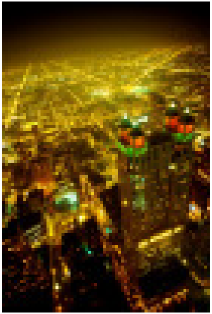


image source: www.istockphoto.com

Problems in Communication, e.g.

Spectrum Sensing [J. Lundén *et al.*,2010],[Moghimi *et al.*,2011], [Wimalajeewa *et al.*,2011]

image source: www.istockphoto.com

Where are we Today?

Advanced Robust Estimation Tasks in Engineering

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Other Areas of Engineering, e.g.

Biomedical [Leski,2002],[Mahadevan *et al.*,2004], [Bénar *et al.*,2007], [Heritier *et al.*,2009],[Liang *et al.*,2009],[Muma *et al.*,2011]

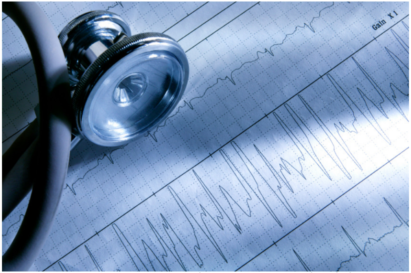

image source: www.istockphoto.com

Other Areas of Engineering, e.g.

Load Forecasting [Mili *et al.*,2002],[Huang *et al.*,2003], [Chakhchoukh *et al.*,2009],[Chakhchoukh,2010]

image source: www.istockphoto.com

The Linear Regression Model

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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Application 2: Localisation of a Mobile User Equipment

image source: www.istockphoto.com

Robust Estimation Application Example

Localisation of a Mobile User Equipment

Application 2: Localisation of a Mobile User Equipment

image source: www.istockphoto.com

- localise wireless transmitter device using different base stations
- important task in many civilian and military applications
- urban scenario: Line-of-Sight (LOS) and Non-Line-of-Sight (NLOS) propagation $\rightarrow$ severe degradation of position estimates

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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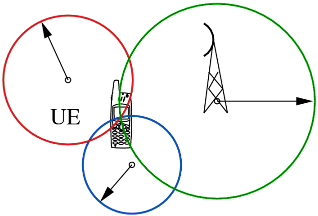
Localisation Based on Time of Arrival (TOA) Measurements:

[Gustafsson *et al.*,2005]



non-linear measurement equation at each base station (BS)

$$y_n = h(\boldsymbol{\theta}) + v_n, \quad n = 1, \ldots, N$$

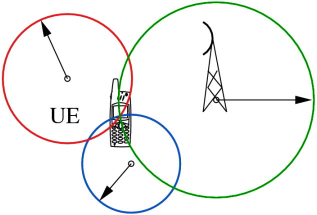
- $h = \sqrt{(x - x_{BS,m})^2 + (x - y_{BS,m})^2}$ distance from the UE to the m-th BS
- $\boldsymbol{\theta} = (x, y)^{\mathsf{T}}$ position of the UE and $m = 1, \ldots, M$

Robust Estimation Application Example

Localisation of a Mobile User Equipment

Localisation Based on Time of Arrival (TOA) Measurements:

[Gustafsson *et al.*,2005]



linearisation yields linear regression model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{E}$$

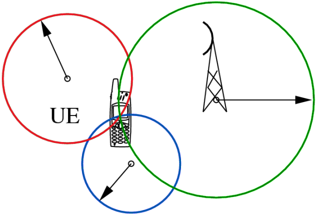
- $\mathbf{X}$ and $\mathbf{E}$ are i.i.d.,$\boldsymbol{\theta}$ is of dimension $p \times 1$
- $\mathbf{Y}(n) = y_n$ and $\mathbf{x}_n'$ is the n^{th} row of the matrix $\mathbf{X}$

Robust Estimation Application Example

Localisation of a Mobile User Equipment

Localisation Based on Time of Arrival (TOA) Measurements:

[Gustafsson *et al.*,2005]



NLOS propagation → outliers in TOA measurements

- possible model

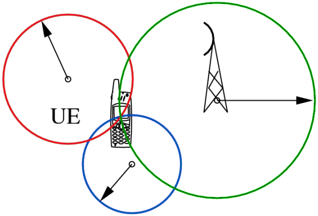
$$f_E(e) = (1 - \varepsilon) f_{LOS}(0, \sigma^2_{LOS}) + \varepsilon f_{NLOS}(\mu_{NLOS}, \sigma^2_{NLOS})$$

Robust Estimation Application Example

Localisation of a Mobile User Equipment

Localisation Based on Time of Arrival (TOA) Measurements:

[Gustafsson *et al.*,2005]



[illegible]ation?

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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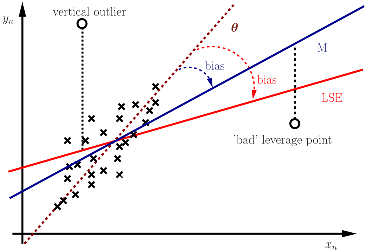
Effect of Outliers



1. Gaussian ML estimator

$$\sum_{n=1}^{N}(\mathbf{y}-\mathbf{x}\hat{\theta})^2=\sum_{n=1}^{N}(\hat{e}_n)^2=\min_{\boldsymbol{\theta}}$$

not robust against any type of outliers

Robust Estimation Application Example

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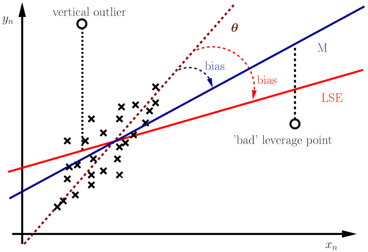
Effect of Outliers



2. M-estimators:

$$\sum_{n=1}^{N} \rho\left(\frac{\hat{e}_n}{\hat{\sigma}_{E,\mathrm{rob}}}\right) = \min_{\boldsymbol{\theta}}$$

robust against vertical outliers

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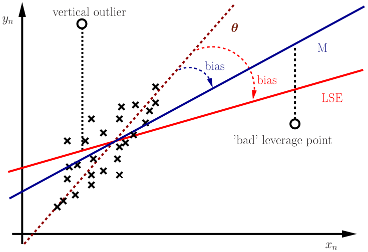
Effect of Outliers



2. M-estimators:

$$\sum_{n=1}^{N} \psi \left(\frac{\hat{e}_n}{\hat{\sigma}_{E,\mathrm{rob}}} \right) \mathbf{x}_n = \mathbf{0}$$

BP=0 for outliers in $\mathbf{x}_n$ ('bad' leverage points)

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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3. S-estimator: [Rousseeuw *et al.*, 1984], [Salibian-Barrera *et al.*, 2006]

   minimizes robust scale of the residuals

$$\widehat{\boldsymbol{\theta}}_S = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\ \hat{\sigma}_{E,\text{rob}}(\boldsymbol{\theta})$$

   however: combination of high efficiency and BP not possible

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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3. S-estimator: [Rousseeuw *et al.*, 1984], [Salibian-Barrera *et al.*, 2006]

minimizes robust scale of the residuals

$$\widehat{\boldsymbol{\theta}}_S = \underset{\boldsymbol{\theta}}{\text{argmin}}\ \hat{\sigma}_{E,\text{rob}}(\boldsymbol{\theta})$$

however: combination of high efficiency and BP not possible

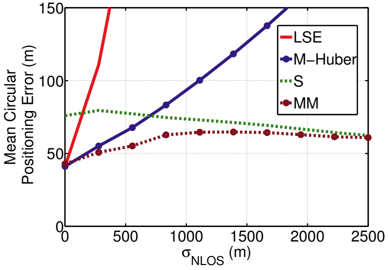
4. MM-estimator: [Yohai, 1987], [Salibian-Barrera *et al.*, 2006]

Step 1. Compute an initial consistent high BP estimate $\hat{\boldsymbol{\theta}}_S$.
Step 2. Compute the high BP M-scale of the residuals of **Step 1**.
Step 3. Compute an efficient M-estimate of regression, using an iterative procedure starting at $\hat{\boldsymbol{\theta}}_S$.

highly robust and efficient: BP=0.5 and Eff=0.95

Simulation Geolocation

- M = 10 base stations
- N = 5 measurements available at each base station
- ε = 0.4
- σ_{LOS} = 150m, $h_{NLOS}(v)$ is the exponential density



MM-Estimator

- outperforms all competitors
- stable performance over all σ_{NLOS}

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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Simulation Geolocation

- M = 10 base stations
- N = 5 measurements available at each base station
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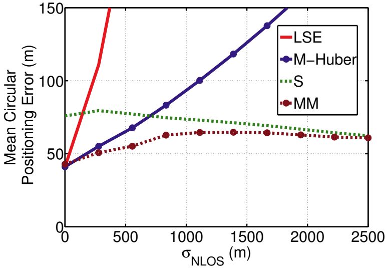

Current Tools [Salibian-Barrera *et al.*, 2006],[Agullo *et al.*, 2008]

fast algorithms to compute robust and efficient estimators

Robust Estimation Application Example

Localisation of a Mobile User Equipment

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Simulation Geolocation

- M = 10 base stations
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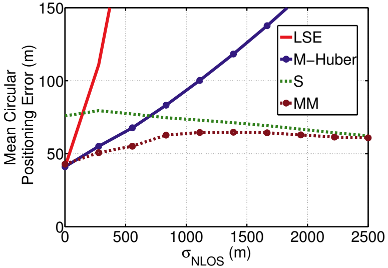

Further Innovations: robust methods that adapt to an unknown scenario, e.g.

U. Hammes and A. M. Zoubir, Robust MT Tracking Based on M–Estimation and Interacting Multiple Model Algorithm., IEEE Trans. Signal Process., Vol. 59, No. 7, pp. 3398–3409, 2011.

F. Yin, C. Fritsche, F. Gustafsson and A. M. Zoubir, TOA Based Robust Wireless Geolocation and Cramer–Rao Lower Bound Analysis in Harsh LOS/NLOS Environments., IEEE Trans. Signal Process., Vol. 61, No. 9, pp. 2243–2255, 2013.

Current Trends and Future Directions

Current Trends and Future Directions

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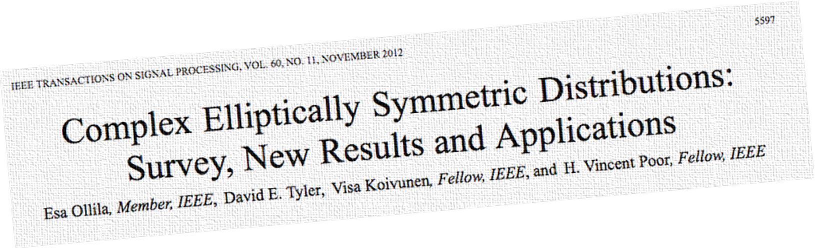
Complex Valued Multichannel Data



5597

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 60, NO. 11, NOVEMBER 2012

Complex Elliptically Symmetric Distributions: Survey, New Results and Applications

Esa Ollila, *Member, IEEE*, David E. Tyler, Visa Koivunen, *Fellow, IEEE*, and H. Vincent Poor, *Fellow, IEEE*

- complex elliptically symmetric distributions
- robust M-estimation for complex valued data
- robust detection of circularity

Current Trends and Future Directions

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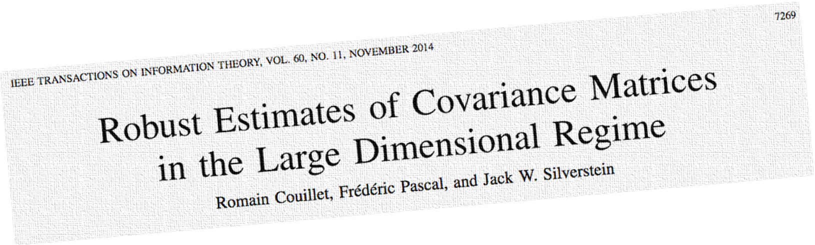
Robustness and Random Matrix Theory


IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 60, NO. 11, NOVEMBER 2014 7269

Robust Estimates of Covariance Matrices in the Large Dimensional Regime

Romain Couillet, Frédéric Pascal, and Jack W. Silverstein

- DOA estimation in the joint $p \to \infty$, $N \to \infty$ regime.
- Adaptive Normalized Matched Filter Detector in the joint $p \to \infty$, $N \to \infty$ regime.

Current Trends and Future Directions

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Regularized Robust Estimation

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 62, NO. 22, NOVEMBER 15, 2014 6059

Regularized M-Estimators of Scatter Matrix

Esa Ollila, *Member, IEEE*, and David E. Tyler

- high-dimensional data $p > n$ containing outliers/impulsive noise
- covariance/scatter matrix estimation

Current Trends and Future Directions

Contaminated regressor models in high dimensional data

Detecting Deviating Data Cells

Peter J. Rousseeuw* and Wannes Van den Bossche
Department of Mathematics, KU Leuven, Belgium

July 6, 2016

Rousseeuw: "Recently researchers have come to realize that the outlying rows paradigm is no longer sufficient for modern high-dimensional datasets."

- high-dimensional regression models containing outliers in the regressors
- Current robust estimators break down in the independent contamination model (ICM)
- New robust lasso procedures are being proposed for the ICM

J. Machkour, B. Alt, M. Muma and A. M. Zoubir, The Outlier-Corrected-Data Adaptive Lasso: A new robust estimator for the independent contamination model.,submitted to ICASSP 2017.

Current Trends and Future Directions

Robust Bootstrap

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 64, NO. 4, FEBRUARY 15, 2016 1007

Robust, Scalable, and Fast Bootstrap Method for Analyzing Large Scale Data

Shahab Basiri, Esa Ollila, *Member, IEEE*, and Visa Koivunen, *Fellow, IEEE*

- Robust bootstrap methods
- Robust bootstrap for big data

Robust Estimation for Dependent Data

Bounded Influence Propagation τ-Estimation: A New Robust Method for ARMA Models

Michael Muma, *Member, IEEE*, and Abdelhak M. Zoubir, *Fellow, IEEE*

- Bounded Influence Propagation ARMA models
- Propagation of outliers

Current Trends and Future Directions

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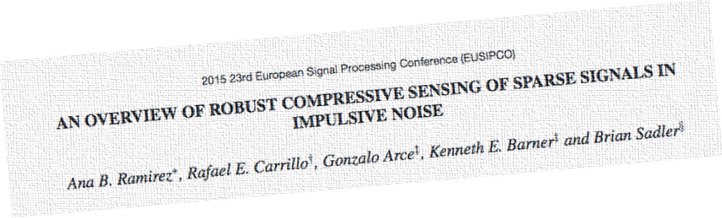
Robust Norms and Compressed Sensing


2015 23rd European Signal Processing Conference (EUSIPCO)

AN OVERVIEW OF ROBUST COMPRESSIVE SENSING OF SPARSE SIGNALS IN IMPULSIVE NOISE

Ana B. Ramirez[*], *Rafael E. Carrillo*[†], *Gonzalo Arce*[‡], *Kenneth E. Barner*[‡] *and Brian Sadler*[§]

- sampling process is performed in the presence of impulsive noise
- robust sampling and nonlinear reconstruction strategies for sparse signals

Current Trends and Future Directions

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Robustness in Distributed and Adaptive Systems


IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 64, NO. 11, JUNE 1, 2016 2851

Robust Adaptation in Impulsive Noise

Sara Al-Sayed, *Student Member, IEEE*, Abdelhak M. Zoubir, *Fellow, IEEE*, and Ali H. Sayed, *Fellow, IEEE*

- Robustness in decentralized sensor networks
- Robust distributed signal and parameter estimation, detection, classification, object labelling, dictionary learning

Current Trends and Future Directions

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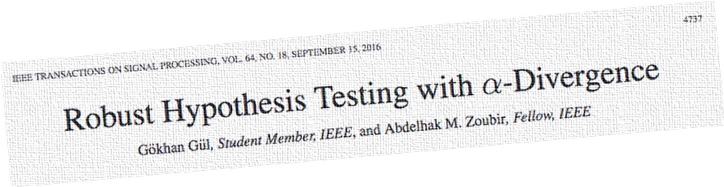
Advances in Robust Detection


4737

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 64, NO. 18, SEPTEMBER 15, 2016

Robust Hypothesis Testing with α-Divergence

Gökhan Gül, *Student Member, IEEE*, and Abdelhak M. Zoubir, *Fellow, IEEE*

- Design of sequential robust detectors
- Joint detection and estimation
- Adaptation of signal models to be usable in today's applications, e.g. radar

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Many of these exciting topics will be covered in this Summer School!

	Sunday 18.9.	Monday, 19.9.	Tuesday, 20.9.	Wednesday, 21.9.	Thursday, 22.9.	Friday, 23.9.	Saturday, 24.9.	
09:00		Gini, Welcome Zoubir, Rob. Basics	Koivunen, Boot. B. Data	Pascal, Rob. RMT	Ollila, Rob.Reg. Cov	Arce, Rob. Norms CS	Special Social Activity (optional)	09:00
10:20		Coffee	Coffee	Coffee	Coffee	Coffee		10:20
10:40		Zoubir, Rob. Regres.	Koivunen, Boot. B. Data	Pascal, Rob. RMT	Ollila, Rob.Reg. Cov	Arce, Rob. Norms CS		10:40
12:00		Lunch	Lunch	Lunch	Lunch	Lunch		12:00
14:00		Muma, Dependent	Koivunen, Boot. B. Data	Pascal, Rob. RMT	Ollila, Rob.Reg. Cov	Arce, Rob. Norms CS		14:00
15:20		Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break		15:20
15:40		Muma, Dependent	Talk to Koivunen	Talk to Pascal	Talk to Ollila	Talk to Arce		15:40
16:10				Poster Session	Social Activity 3 (optional)	Exam (optional)		16:10
			Social Activity 2 (optional)					
17:00		Talk to Zoubir, Muma						17:00
17:30								17:30
19:00	Welcome Dinner	Social Activity 1 (optional)						19:00
				Special Dinner				
21:00			Chill-Out Vineyard (optional)					21:00
23:00								23:00

Thanks for your attention!

Literature

Further Reading Robustness in Signal Processing I

TECHNISCHE UNIVERSITÄT DARMSTADT

[1] F. R. Hampel, E. M. Ronchetti, P. J. Rousseeuw and W. A. Stahel,
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